

Mid-Chapter Quiz: Lessons 8-1 through 8-4

Find the geometric mean between each pair of numbers.

1. 12 and 3

SOLUTION:

The geometric mean between a and b is given by, $x = \sqrt{ab}$.
Substitute 12 for a and 3 for b .

$$\begin{aligned}x &= \sqrt{12 \cdot 3} \\&= \sqrt{36} \\&= 6\end{aligned}$$

ANSWER:

6

2. 63 and 7

SOLUTION:

The geometric mean between a and b is given by, $x = \sqrt{ab}$.
Substitute 63 for a and 7 for b .

$$\begin{aligned}x &= \sqrt{63 \cdot 7} \\&= \sqrt{441} \\&= 21\end{aligned}$$

ANSWER:

21

3. 45 and 20

SOLUTION:

The geometric mean between a and b is given by, $x = \sqrt{ab}$.
Substitute 45 for a and 20 for b .

$$\begin{aligned}x &= \sqrt{45 \cdot 20} \\&= \sqrt{900} \\&= 30\end{aligned}$$

ANSWER:

30

4. 50 and 10

SOLUTION:

The geometric mean between a and b is given by, $x = \sqrt{ab}$.
Substitute 50 for a and 10 for b .

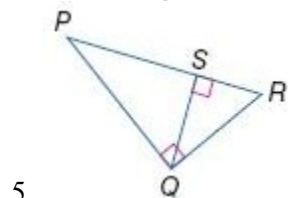
$$\begin{aligned}x &= \sqrt{50 \cdot 10} \\&= \sqrt{500} \\&= \sqrt{100 \cdot 5} \\&= 10\sqrt{5}\end{aligned}$$

ANSWER:

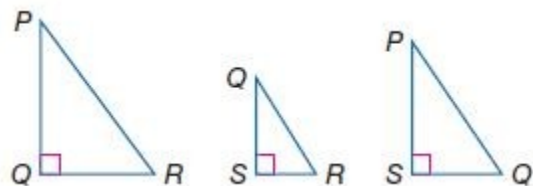
$10\sqrt{5}$

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Write a similarity statement identifying the three similar triangles in each figure.



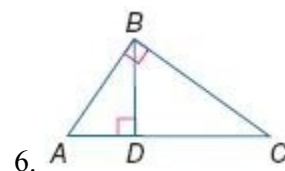
SOLUTION:



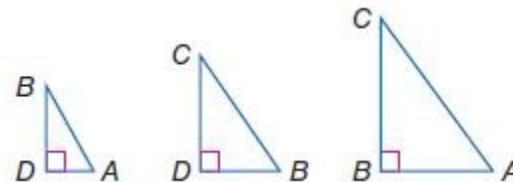
$$\triangle PRQ \sim \triangle QRS \sim \triangle PQS$$

ANSWER:

$$\triangle PRQ \sim \triangle QRS \sim \triangle PQS$$



SOLUTION:

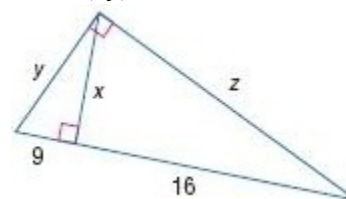


$$\triangle ABD \sim \triangle BCD \sim \triangle ACB$$

ANSWER:

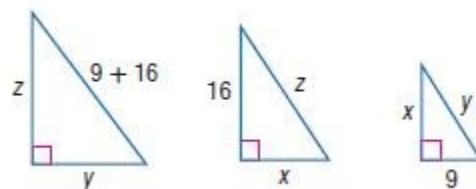
$$\triangle ABD \sim \triangle BCD \sim \triangle ACB$$

Find x , y , and z .



7.

SOLUTION:



By the Geometric Mean (Leg) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of a leg of this triangle is the geometric mean

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between the length of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

$$\frac{\text{hypotenuse}}{\text{longer leg}} = \frac{\text{hypotenuse}}{\text{longer leg}} \quad \frac{\text{hypotenuse}}{\text{shorter leg}} = \frac{\text{hypotenuse}}{\text{shorter leg}}$$

Set up a proportion and solve for z and y :

$$\begin{aligned} \frac{25}{z} &= \frac{z}{16} & \text{and} & \quad \frac{25}{y} = \frac{y}{9} \\ z^2 &= 400 & y^2 &= 225 \\ z &= \sqrt{400} & \text{and} & \quad y = \sqrt{225} \\ &= 20 & & \quad = 15 \end{aligned}$$

By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of this altitude is the geometric mean between the lengths of these two segments.

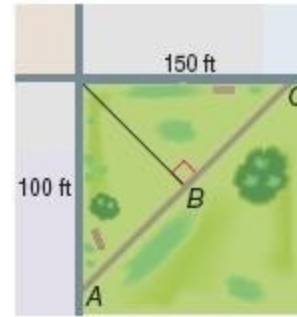
Solve for x :

$$\begin{aligned} \frac{16}{x} &= \frac{x}{9} \\ x^2 &= 144 \\ x &= 12 \end{aligned}$$

ANSWER:

$$x = 12, y = 15, z = 20$$

8. **PARKS** There is a small park in a corner made by two perpendicular streets. The park is 100 ft by 150 ft, with a diagonal path, as shown below. What is the length of path \overline{AC} ?



SOLUTION:

Use Pythagorean theorem to find the length of the path \overline{AC} .

$$\begin{aligned} 100^2 + 150^2 &= AC^2 \\ 10000 + 22500 &= AC^2 \\ 32500 &= AC^2 \\ \sqrt{32500} &= AC \\ 180.3 &\approx AC \end{aligned}$$

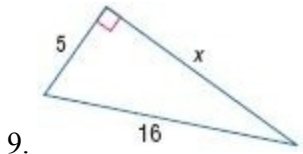
The length of path \overline{AC} is 180.3 ft.

ANSWER:

$$180.3 \text{ ft}$$

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Find x . Round to the nearest hundredth.



SOLUTION:

Use Pythagorean theorem to find the length x .

$$x^2 + 5^2 = 16^2$$

$$x^2 + 25 = 256$$

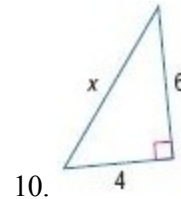
$$x^2 = 231$$

$$x = \sqrt{231}$$

$$x \approx 15.20$$

ANSWER:

$$\sqrt{231} \approx 15.20$$



SOLUTION:

Use Pythagorean theorem to find the length of the hypotenuse, x .

$$4^2 + 6^2 = x^2$$

$$16 + 36 = x^2$$

$$52 = x^2$$

$$\sqrt{52} = x$$

$$x = 2\sqrt{13} \approx 7.21$$

ANSWER:

$$2\sqrt{13} \approx 7.21$$

11. **MULTIPLE CHOICE** Which of the following sets of numbers is not a Pythagorean triple?

A 9, 12, 15

B 21, 72, 75

C 15, 36, 39

D 8, 13, 15

SOLUTION:

By Pythagorean theorem,

$a^2 + b^2 = c^2$, where c is the hypotenuse of the right triangle.

In choice **A**,

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$$9^2 + 12^2 = 15^2$$

$$81 + 144 = 225$$

$$225 = 225 \quad \checkmark$$

In choice **B**,

$$21^2 + 72^2 = 75^2$$

$$441 + 5184 = 5625$$

$$5625 = 5625 \quad \checkmark$$

In choice **C**,

$$15^2 + 36^2 = 39^2$$

$$225 + 1296 = 1521$$

$$1521 = 1521 \quad \checkmark$$

In choice **D**,

$$8^2 + 13^2 = 15^2$$

$$64 + 169 = 225$$

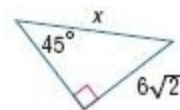
$$233 = 225 \quad \times$$

Therefore, the correct choice is **D**.

ANSWER:

D

Find x .



12.

SOLUTION:

Use special right triangles to find the value of x . In 45-45-90 triangles, the hypotenuse is $\sqrt{2}$ times the leg lengths (l), which are equal.

Therefore, since the legs of this triangle are $6\sqrt{2}$, the hypotenuse would be $h = (6\sqrt{2})\sqrt{2}$.

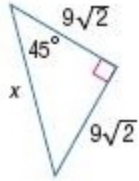
Simplify:

$$\begin{aligned} h &= (6\sqrt{2})\sqrt{2} \\ &= 6(\sqrt{2} \cdot \sqrt{2}) \\ &= 6 \cdot 2 \\ &= 12 \end{aligned}$$

ANSWER:

12

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13.

SOLUTION:

Use special right triangles to find the value of x . In 45-45-90 triangles, the hypotenuse is $\sqrt{2}$ times the leg lengths (l), which are equal.

Therefore, since the legs of this triangle are $9\sqrt{2}$, the hypotenuse would be $h = (9\sqrt{2})\sqrt{2}$.

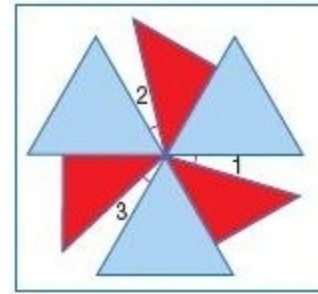
Simplify:

$$\begin{aligned} h &= (9\sqrt{2})\sqrt{2} \\ &= 9(\sqrt{2} \cdot \sqrt{2}) \\ &= 9 \cdot 2 \\ &= 18 \end{aligned}$$

ANSWER:

18

14. **DESIGN** Jamie designed a pinwheel to put in her garden. In the pinwheel, the blue triangles are congruent equilateral triangles, each with an altitude of 4 inches. The red triangles are congruent isosceles right triangles. The hypotenuse of a red triangle is congruent to a side of the blue triangle.



- If angles 1, 2, and 3 are congruent, find the measure of each angle.
- Find the perimeter of the pinwheel.

SOLUTION:

a) Since the base angles of an isosceles triangle are congruent, the measure of each acute angle is 45° . The three angles of an equilateral triangle are congruent and each angle has a measure of 60° .

Let x represent the $m\angle 1$, $m\angle 2$, or $m\angle 3$. Set all the measures of all the angles in the pinwheel equal to 360 and solve for x :

$$\begin{aligned} 3(x) + 3(45) + 3(60) &= 360 \\ 3x + 135 + 180 &= 360 \\ 3x + 315 &= 360 \\ 3x &= 45 \\ x &= 15 \end{aligned}$$

Therefore, the angles 1, 2, and 3 each have a measure of 15° .

b) The altitude of an equilateral triangle divides the triangle into two 30-60-90 right triangles with a side of the equilateral triangle as the hypotenuse. The altitude (the longer leg) is 4 inches. If s is the shortest side of the 30-60-90 triangle, then the length of the hypotenuse is $2s$ and the length of the longer leg is $s\sqrt{3}$.

Solve for s :

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$$\begin{aligned}s\sqrt{3} &= 4 \\ s &= \frac{4}{\sqrt{3}} \\ &= \frac{4\sqrt{3}}{3}\end{aligned}$$

The length of the hypotenuse (and the side of the equilateral triangle) is

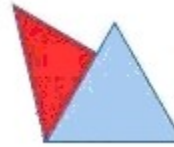
$$2\left(\frac{8\sqrt{3}}{3}\right) = \frac{8\sqrt{3}}{3}.$$

The red triangles are isosceles triangles with hypotenuse measuring $\frac{8\sqrt{3}}{3}$.

So, the length of each leg of the red triangle is:

$$\begin{aligned}\frac{\frac{8\sqrt{3}}{3}}{\sqrt{2}} &= \frac{8\sqrt{3}}{3} \cdot \frac{1}{\sqrt{2}} \\ &= \frac{8\sqrt{3}}{3\sqrt{2}} \\ &= \frac{8\sqrt{3}}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{8\sqrt{6}}{6} \text{ or } \frac{4\sqrt{6}}{3}.\end{aligned}$$

Find the perimeter of the one of the wings.



$$\text{Perimeter of this figure} \left\} = \frac{8\sqrt{3}}{3} + \frac{8\sqrt{3}}{3} + \left(\frac{8\sqrt{3}}{3} - \frac{4\sqrt{6}}{3}\right) + \frac{4\sqrt{6}}{3} + \frac{8\sqrt{3}}{3}$$

Simplify.

$$\begin{aligned}&= \frac{8\sqrt{3}}{3} + \frac{8\sqrt{3}}{3} + \frac{8\sqrt{3}}{3} + \frac{8\sqrt{3}}{3} \\ &= 4 \cdot \frac{8\sqrt{3}}{3} \\ &= \frac{32\sqrt{3}}{3}\end{aligned}$$

There are 3 such wings, therefore the perimeter of the pinwheel is

$$3 \cdot \frac{32\sqrt{3}}{3} = 32\sqrt{3} \approx 55.$$

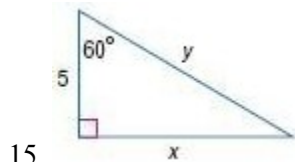
The perimeter of the pinwheel is about 55 inches.

ANSWER:

- a. 15
- b. 55 in.

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Find x and y .



SOLUTION:

In 30-60-90 right triangles, the hypotenuse is twice the shorter leg ($h = 2s$), and the longer leg is $\sqrt{3}$ times the shorter leg ($l = s\sqrt{3}$).

Solve for x :

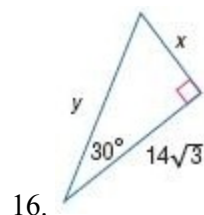
$$x = 5\sqrt{3}$$

Solve for y :

$$y = 2(5) = 10$$

ANSWER:

$$x = 5\sqrt{3}; y = 10$$



SOLUTION:

In 30-60-90 right triangles, the hypotenuse is twice the shorter leg ($h = 2s$), and the longer leg is $\sqrt{3}$ times the shorter leg ($l = s\sqrt{3}$).

Solve for x :

$$x\sqrt{3} = 14\sqrt{3}$$

$$x = 14$$

Solve for y :

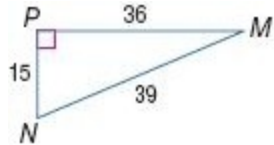
$$y = 2(14) = 28$$

ANSWER:

$$x = 14; y = 28$$

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Express each ratio as a fraction and as a decimal to the nearest hundredth.



17. $\tan M$

SOLUTION:

$$\begin{aligned}\tan M &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{15}{36} \\ &\approx 0.42\end{aligned}$$

ANSWER:

$$\frac{15}{36} = 0.42$$

18. $\cos M$

SOLUTION:

$$\begin{aligned}\cos M &= \frac{\text{adjacent}}{\text{hypotenuse}} \\ &= \frac{36}{39} \\ &\approx 0.92\end{aligned}$$

ANSWER:

$$\frac{36}{39} = 0.92$$

19. $\cos N$

SOLUTION:

$$\begin{aligned}\cos N &= \frac{\text{adjacent}}{\text{hypotenuse}} \\ &= \frac{15}{39} \\ &\approx 0.38\end{aligned}$$

ANSWER:

$$\frac{15}{39} \approx 0.38$$

20. $\sin N$

SOLUTION:

$$\begin{aligned}\sin N &= \frac{\text{opposite}}{\text{hypotenuse}} \\ &= \frac{36}{39} \\ &\approx 0.92\end{aligned}$$

ANSWER:

$$\frac{36}{39} \approx 0.92$$

21. Solve the right triangle. Round angle measures to the nearest degree and side measures to the nearest tenth.



SOLUTION:

Use Pythagorean theorem to find GJ .

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$$21^2 + 36^2 = GJ^2$$

$$441 + 1296 = GJ^2$$

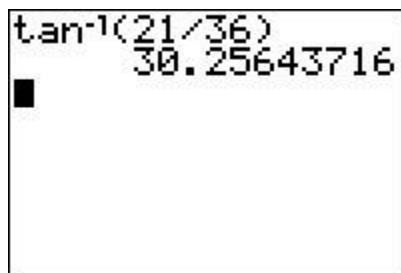
$$GJ = \sqrt{1737}$$

$$GJ \approx 41.7$$

Use the tangent ratio to find $m\angle G$ and $m\angle J$.

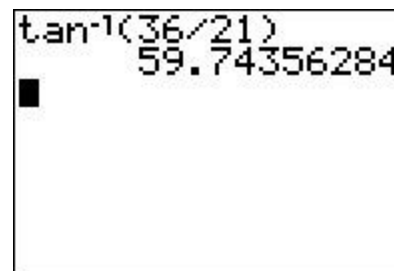
$$\tan G = \frac{\text{opposite}}{\text{adjacent}} = \frac{21}{36}$$

$$G = \tan^{-1}\left(\frac{21}{36}\right)$$
$$\approx 30^\circ$$

A calculator screen with a black border. The display shows the text "tan^-1(21/36)" on the top line and the numerical result "30.25643716" on the line below it. A small black cursor icon is visible to the left of the text.

$$\tan J = \frac{\text{opposite}}{\text{adjacent}} = \frac{36}{21}$$

$$J = \tan^{-1}\left(\frac{36}{21}\right)$$
$$\approx 60^\circ$$

A calculator screen with a black border. The display shows the text "tan^-1(36/21)" on the top line and the numerical result "59.74356284" on the line below it. A small black cursor icon is visible to the left of the text.

ANSWER:

$$JG = 41.7; m\angle G = 30^\circ; m\angle J = 60^\circ$$